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Physics

for the IB Diploma

SIXTH EDITION

K. A. Tsokos

with additional
online material



Physics

for the IB Diploma

Sixth Edition

K. A. Tsokos

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Option D Astrophysics

Additional Topic questions to accompany coursebook

Detailed answers to all coursebook test yourself questions

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Nature of Science

Answers to exam-style questions

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Introduction

This sixth edition of *Physics for the IB Diploma* is fully updated to cover the content of the IB Physics Diploma syllabus that will be examined in the years 2016–2022.

Physics may be studied at Standard Level (SL) or Higher Level (HL). Both share a common core, which is covered in [Topics 1–8](#). At HL the core is extended to include [Topics 9–12](#). In addition, at both levels, students then choose one Option to complete their studies. Each option consists of common core and additional Higher Level material. You can identify the HL content in this book by ‘HL’ included in the topic title (or section title in the Options), and by the red page border. The four Options are included in the free online material that is accessible using education.cambridge.org/ibsciences.

The structure of this book follows the structure of the IB Physics syllabus. Each topic in the book matches a syllabus topic, and the sections within each topic mirror the sections in the syllabus. Each section begins with learning objectives as starting and reference points. Worked examples are included in each section; understanding these examples is crucial to performing well in the exam. A large number of test yourself questions are included at the end of each section and each topic ends with exam-style questions. The reader is strongly encouraged to do as many of these questions as possible. Numerical answers to the test yourself questions are provided at the end of the book; detailed solutions to all questions are available on the website. Some topics have additional questions online; these are indicated with the online symbol, shown here.

Theory of Knowledge (TOK) provides a cross-curricular link between different subjects. It stimulates thought about critical thinking and how we can say we know what we claim to know. Throughout this book, TOK features highlight concepts in Physics that can be considered from a TOK perspective. These are indicated by the ‘TOK’ logo, shown here.

Science is a truly international endeavour, being practised across all continents, frequently in international or even global partnerships. Many problems that science aims to solve are international, and will require globally implemented solutions. Throughout this book, International-Mindedness features highlight international concerns in Physics. These are indicated by the ‘International-Mindedness’ logo, shown here.

Nature of science is an overarching theme of the Physics course. The theme examines the processes and concepts that are central to scientific endeavour, and how science serves and connects with the wider community. At the end of each section in this book, there is a ‘Nature of science’ paragraph that discusses a particular concept or discovery from the point of view of one or more aspects of Nature of science. A chapter giving a general introduction to the Nature of science theme is available in the free online material.



Free online material

Additional material to support the IB Physics Diploma course is available online. Visit education.cambridge.org/ibsciences and register to access these resources.

Besides the Options and Nature of science chapter, you will find a collection of resources to help with revision and exam preparation. This includes guidance on the assessments, additional Topic questions, interactive self-test questions and model examination papers and mark schemes. Additionally, answers to the exam-style questions in this book and to all the questions in the Options are available.

Note from the author

This book is dedicated to Alexios and Alkeos and to the memory of my parents.

I have received help from a number of students at ACS Athens in preparing some of the questions included in this book. These include Konstantinos Damianakis, Philip Minaretzis, George Nikolakoudis, Katayoon Khoshragham, Kyriakos Petrakos, Majdi Samad, Stavroula Stathopoulou, Constantine Tragakes and Rim Versteeg. I sincerely thank them all for the invaluable help.

I owe an enormous debt of gratitude to Anne Trevillion, the editor of the book, for her patience, her attention to detail and for the very many suggestions she made that have improved the book substantially. Her involvement with this book exceeded the duties one ordinarily expects from an editor of a book and I thank her from my heart. I also wish to thank her for her additional work of contributing to the Nature of science themes throughout the book.

Finally, I wish to thank my wife, Ellie Tragakes, for her patience with me during the completion of this book.

K.A. Tsokos

Measurement and uncertainties 1

1.1 Measurement in physics

Physics is an experimental science in which measurements made must be expressed in units. In the international system of units used throughout this book, the SI system, there are seven fundamental units, which are defined in this section. All quantities are expressed in terms of these units directly, or as a combination of them.

The SI system

The SI system (short for *Système International d'Unités*) has seven **fundamental units** (it is quite amazing that only seven are required).

These are:

- 1 The **metre** (m). This is the unit of distance. It is the distance travelled by light in a vacuum in a time of $\frac{1}{299\,792\,458}$ seconds.
- 2 The **kilogram** (kg). This is the unit of mass. It is the mass of a certain quantity of a platinum–iridium alloy kept at the Bureau International des Poids et Mesures in France.
- 3 The **second** (s). This is the unit of time. A second is the duration of 9 192 631 770 full oscillations of the electromagnetic radiation emitted in a transition between the two hyperfine energy levels in the ground state of a caesium-133 atom.
- 4 The **ampere** (A). This is the unit of electric current. It is defined as that current which, when flowing in two parallel conductors 1 m apart, produces a force of 2×10^{-7} N on a length of 1 m of the conductors.
- 5 The **kelvin** (K). This is the unit of temperature. It is $\frac{1}{273.16}$ of the thermodynamic temperature of the triple point of water.
- 6 The **mole** (mol). One mole of a substance contains as many particles as there are atoms in 12 g of carbon-12. This special number of particles is called Avogadro's number and is approximately 6.02×10^{23} .
- 7 The **candela** (cd). This is a unit of luminous intensity. It is the intensity of a source of frequency 5.40×10^{14} Hz emitting $\frac{1}{683}$ W per steradian.

You do not need to memorise the details of these definitions.

In this book we will use all of the basic units except the last one.

Physical quantities other than those above have units that are combinations of the seven fundamental units. They have **derived** units. For example, speed has units of distance over time, metres per second (i.e. m/s or, preferably, ms^{-1}). Acceleration has units of metres per second squared (i.e. m/s^2 , which we write as ms^{-2}). Similarly, the unit of force is the newton (N). It equals the combination kgms^{-2} . Energy, a very important quantity in physics, has the joule (J) as its unit. The joule is the combination Nm and so equals $(\text{kgms}^{-2}\text{m})$, or $\text{kgm}^2\text{s}^{-2}$. The quantity

Learning objectives

- State the fundamental units of the SI system.
- Be able to express numbers in scientific notation.
- Appreciate the order of magnitude of various quantities.
- Perform simple order-of-magnitude calculations mentally.
- Express results of calculations to the correct number of significant figures.

power has units of energy per unit of time, and so is measured in J s^{-1} . This combination is called a watt. Thus:

$$1 \text{ W} = (1 \text{ N m s}^{-1}) = (1 \text{ kg m s}^{-2} \text{ m s}^{-1}) = 1 \text{ kg m}^2 \text{ s}^{-3}$$

Metric multipliers

Small or large quantities can be expressed in terms of units that are related to the basic ones by powers of 10. Thus, a nanometre (nm) is 10^{-9} m, a microgram (μg) is 10^{-6} g = 10^{-9} kg, a gigaelectron volt (GeV) equals 10^9 eV, etc. The most common prefixes are given in Table 1.1.

Power	Prefix	Symbol	Power	Prefix	Symbol
10^{-18}	atto-	A	10^1	deka-	da
10^{-15}	femto-	F	10^2	hecto-	h
10^{-12}	pico-	p	10^3	kilo-	k
10^{-9}	nano-	n	10^6	mega-	M
10^{-6}	micro-	μ	10^9	giga-	G
10^{-3}	milli-	m	10^{12}	tera-	T
10^{-2}	centi-	c	10^{15}	peta-	P
10^{-1}	deci-	d	10^{18}	exa-	E

Table 1.1 Common prefixes in the SI system.

Orders of magnitude and estimates

Expressing a quantity as a plain power of 10 gives what is called the **order of magnitude** of that quantity. Thus, the mass of the universe has an order of magnitude of 10^{53} kg and the mass of the Milky Way galaxy has an order of magnitude of 10^{41} kg. The ratio of the two masses is then simply 10^{12} .

Tables 1.2, 1.3 and 1.4 give examples of distances, masses and times, given as orders of magnitude.

	Length / m
distance to edge of observable universe	10^{26}
distance to the Andromeda galaxy	10^{22}
diameter of the Milky Way galaxy	10^{21}
distance to nearest star	10^{16}
diameter of the solar system	10^{13}
distance to the Sun	10^{11}
radius of the Earth	10^7
size of a cell	10^{-5}
size of a hydrogen atom	10^{-10}
size of an $A = 50$ nucleus	10^{-15}
size of a proton	10^{-15}
Planck length	10^{-35}

Table 1.2 Some interesting distances.



	Mass / kg
the universe	10^{53}
the Milky Way galaxy	10^{41}
the Sun	10^{30}
the Earth	10^{24}
Boeing 747 (empty)	10^5
an apple	0.2
a raindrop	10^{-6}
a bacterium	10^{-15}
smallest virus	10^{-21}
a hydrogen atom	10^{-27}
an electron	10^{-30}

Table 1.3 Some interesting masses.

	Time / s
age of the universe	10^{17}
age of the Earth	10^{17}
time of travel by light to nearby star	10^8
one year	10^7
one day	10^5
period of a heartbeat	1
lifetime of a pion	10^{-8}
lifetime of the omega particle	10^{-10}
time of passage of light across a proton	10^{-24}

Table 1.4 Some interesting times.

Worked examples

1.1 Estimate how many grains of sand are required to fill the volume of the Earth. (This is a classic problem that goes back to Aristotle. The radius of the Earth is about 6×10^6 m.)

The volume of the Earth is:

$$\frac{4}{3}\pi R^3 \approx \frac{4}{3} \times 3 \times (6 \times 10^6)^3 \approx 8 \times 10^{20} \approx 10^{21} \text{ m}^3$$

The diameter of a grain of sand varies of course, but we will take 1 mm as a fair estimate. The volume of a grain of sand is about $(1 \times 10^{-3})^3 \text{ m}^3$.

Then the number of grains of sand required to fill the Earth is:

$$\frac{10^{21}}{(1 \times 10^{-3})^3} \approx 10^{30}$$

1.2 Estimate the speed with which human hair grows.

I have my hair cut every two months and the barber cuts a length of about 2 cm. The speed is therefore:

$$\begin{aligned} \frac{2 \times 10^{-2}}{2 \times 30 \times 24 \times 60 \times 60} \text{ m s}^{-1} &\approx \frac{10^{-2}}{3 \times 2 \times 36 \times 10^4} \\ &\approx \frac{10^{-6}}{6 \times 40} = \frac{10^{-6}}{240} \\ &\approx 4 \times 10^{-9} \text{ m s}^{-1} \end{aligned}$$

1.3 Estimate how long the line would be if all the people on Earth were to hold hands in a straight line. Calculate how many times it would wrap around the Earth at the equator. (The radius of the Earth is about 6×10^6 m.)

Assume that each person has his or her hands stretched out to a distance of 1.5 m and that the population of Earth is 7×10^9 people.

Then the length of the line of people would be $7 \times 10^9 \times 1.5 \text{ m} = 10^{10} \text{ m}$.

The circumference of the Earth is $2\pi R \approx 6 \times 6 \times 10^6 \text{ m} \approx 4 \times 10^7 \text{ m}$.

So the line would wrap $\frac{10^{10}}{4 \times 10^7} \approx 250$ times around the equator.

1.4 Estimate how many apples it takes to have a combined mass equal to that of an ordinary family car.

Assume that an apple has a mass of 0.2 kg and a car has a mass of 1400 kg.

Then the number of apples is $\frac{1400}{0.2} = 7 \times 10^3$.

1.5 Estimate the time it takes light to arrive at Earth from the Sun. (The Earth–Sun distance is 1.5×10^{11} m.)

The time taken is $\frac{\text{distance}}{\text{speed}} = \frac{1.5 \times 10^{11}}{3 \times 10^8} \approx 0.5 \times 10^4 = 500 \text{ s} \approx 8 \text{ min}$

Significant figures

The **number** of digits used to express a number carries information about how precisely the number is known. A stopwatch reading of 3.2 s (two significant figures, s.f.) is less precise than a reading of 3.23 s (three s.f.). If you are told what your salary is going to be, you would like that number to be known as precisely as possible. It is less satisfying to be told that your salary will be ‘about 1000’ (1 s.f.) euro a month compared to a salary of ‘about 1250’ (3 s.f.) euro a month. Not because 1250 is larger than 1000 but because the number of ‘about 1000’ could mean anything from a low of 500 to a high of 1500. You could be lucky and get the 1500 but you cannot be sure. With a salary of ‘about 1250’ your actual salary could be anything from 1200 to 1300, so you have a pretty good idea of what it will be.

How to find the number of significant figures in a number is illustrated in [Table 1.5](#).



Number	Number of s.f.	Reason	Scientific notation
504	3	in an integer all digits count (if last digit is not zero)	5.04×10^2
608 000	3	zeros at the end of an integer do not count	6.08×10^5
200	1	zeros at the end of an integer do not count	2×10^2
0.000 305	3	zeros in front do not count	3.05×10^{-4}
0.005 900	4	zeros at the end of a decimal count, those in front do not	5.900×10^{-3}

Table 1.5 Rules for significant figures.

Scientific notation means writing a number in the form $a \times 10^b$, where a is decimal such that $1 \leq a < 10$ and b is a positive or negative integer. The number of digits in a is the number of significant figures in the number.

In multiplication or division (or in raising a number to a power or taking a root), the result must have as many significant figures as the **least** precisely known number entering the calculation. So we have that:

$$\underbrace{23}_{2 \text{ s.f.}} \times \underbrace{578}_{3 \text{ s.f.}} = 13\,294 \approx \underbrace{1.3 \times 10^4}_{2 \text{ s.f.}} \quad (\text{the least number of s.f. is shown in red})$$

$$\frac{\underbrace{6.244}_{4 \text{ s.f.}}}{\underbrace{1.25}_{3 \text{ s.f.}}} = 4.9952\dots \approx \underbrace{5.00 \times 10^0}_{3 \text{ s.f.}} = 5.00$$

$$\underbrace{12.3^3}_{3 \text{ s.f.}} = 1860.867\dots \approx \underbrace{1.86 \times 10^3}_{3 \text{ s.f.}}$$

$$\sqrt{\underbrace{58900}_{3 \text{ s.f.}}} = 242.6932\dots \approx \underbrace{2.43 \times 10^2}_{3 \text{ s.f.}}$$

In adding and subtracting, the number of decimal digits in the answer must be equal to the least number of decimal places in the numbers added or subtracted. Thus:

$$\underbrace{3.21}_{2 \text{ d.p.}} + \underbrace{4.1}_{1 \text{ d.p.}} = 7.32 \approx \underbrace{7.3}_{1 \text{ d.p.}} \quad (\text{the least number of d.p. is shown in red})$$

$$\underbrace{12.367}_{3 \text{ d.p.}} - \underbrace{3.15}_{2 \text{ d.p.}} = 9.217 \approx \underbrace{9.22}_{2 \text{ d.p.}}$$

Use the rules for rounding when writing values to the correct number of decimal places or significant figures. For example, the number $542.48 = 5.4248 \times 10^2$ rounded to 2, 3 and 4 s.f. becomes:

$$\begin{aligned} 5.4|248 \times 10^2 &\approx 5.4 \times 10^2 && \text{rounded to 2 s.f.} \\ 5.42|48 \times 10^2 &\approx 5.42 \times 10^2 && \text{rounded to 3 s.f.} \\ 5.424|8 \times 10^2 &\approx 5.425 \times 10^2 && \text{rounded to 4 s.f.} \end{aligned}$$

There is a special rule for rounding when the last digit to be dropped is 5 and it is followed only by zeros, or not followed by any other digit.

This is the odd–even rounding rule. For example, consider the number 3.250 000 0... where the zeros continue indefinitely. How does this number round to 2 s.f.? Because the digit before the 5 is **even** we do not round up, so 3.250 000 0... becomes 3.2. But 3.350 000 0... rounds up to 3.4 because the digit before the 5 is **odd**.

Nature of science

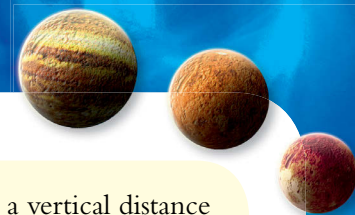
Early work on electricity and magnetism was hampered by the use of different systems of units in different parts of the world. Scientists realised they needed to have a common system of units in order to learn from each other's work and reproduce experimental results described by others. Following an international review of units that began in 1948, the SI system was introduced in 1960. At that time there were six base units. In 1971 the mole was added, bringing the number of base units to the seven in use today.

As the instruments used to measure quantities have developed, the definitions of standard units have been refined to reflect the greater precision possible. Using the transition of the caesium-133 atom to measure time has meant that smaller intervals of time can be measured accurately. The SI system continues to evolve to meet the demands of scientists across the world. Increasing precision in measurement allows scientists to notice smaller differences between results, but there is always uncertainty in any experimental result. There are no 'exact' answers.



Test yourself

- 1 How long does light take to travel across a proton?
- 2 How many hydrogen atoms does it take to make up the mass of the Earth?
- 3 What is the age of the universe expressed in units of the Planck time?
- 4 How many heartbeats are there in the lifetime of a person (75 years)?
- 5 What is the mass of our galaxy in terms of a solar mass?
- 6 What is the diameter of our galaxy in terms of the astronomical unit, i.e. the distance between the Earth and the Sun ($1 \text{ AU} = 1.5 \times 10^{11} \text{ m}$)?
- 7 The molar mass of water is 18 g mol^{-1} . How many molecules of water are there in a glass of water (mass of water 300 g)?
- 8 Assuming that the mass of a person is made up entirely of water, how many molecules are there in a human body (of mass 60 kg)?
- 9 Give an order-of-magnitude estimate of the density of a proton.
- 10 How long does light take to traverse the diameter of the solar system?
- 11 An electron volt (eV) is a unit of energy equal to $1.6 \times 10^{-19} \text{ J}$. An electron has a kinetic energy of 2.5 eV.
 - a How many joules is that?
 - b What is the energy in eV of an electron that has an energy of $8.6 \times 10^{-18} \text{ J}$?
- 12 What is the volume in cubic metres of a cube of side 2.8 cm?
- 13 What is the side in metres of a cube that has a volume of 588 cubic millimetres?
- 14 Give an order-of-magnitude estimate for the mass of:
 - a an apple
 - b this physics book
 - c a soccer ball.



- 15 A white dwarf star has a mass about that of the Sun and a radius about that of the Earth. Give an order-of-magnitude estimate of the density of a white dwarf.
- 16 A sports car accelerates from rest to 100 km per hour in 4.0 s. What fraction of the acceleration due to gravity is the car's acceleration?
- 17 Give an order-of-magnitude estimate for the number of electrons in your body.
- 18 Give an order-of-magnitude estimate for the ratio of the electric force between two electrons 1 m apart to the gravitational force between the electrons.
- 19 The frequency f of oscillation (a quantity with units of inverse seconds) of a mass m attached to a spring of spring constant k (a quantity with units of force per length) is related to m and k . By writing $f = cm^x k^y$ and matching units on both sides, show that $f = c\sqrt{\frac{k}{m}}$, where c is a dimensionless constant.
- 20 A block of mass 1.2 kg is raised a vertical distance of 5.55 m in 2.450 s. Calculate the power delivered. ($P = \frac{mgh}{t}$ and $g = 9.81 \text{ m s}^{-2}$)
- 21 Find the kinetic energy ($E_K = \frac{1}{2}mv^2$) of a block of mass 5.00 kg moving at a speed of 12.5 m s^{-1} .
- 22 Without using a calculator, **estimate** the value of the following expressions. Then compare your estimate with the exact value found using a calculator.
- a $\frac{243}{43}$
- b 2.80×1.90
- c $312 \times \frac{480}{160}$
- d $\frac{8.99 \times 10^9 \times 7 \times 10^{-16} \times 7 \times 10^{-6}}{(8 \times 10^2)^2}$
- e $\frac{6.6 \times 10^{-11} \times 6 \times 10^{24}}{(6.4 \times 10^6)^2}$

1.2 Uncertainties and errors

This section introduces the basic methods of dealing with experimental error and uncertainty in measured physical quantities. Physics is an experimental science and often the experimenter will perform an experiment to test the prediction of a given theory. No measurement will ever be completely accurate, however, and so the result of the experiment will be presented with an experimental error.

Types of uncertainty

There are two main types of uncertainty or error in a measurement. They can be grouped into **systematic** and **random**, although in many cases it is not possible to distinguish clearly between the two. We may say that random uncertainties are almost always the fault of the observer, whereas systematic errors are due to both the observer and the instrument being used. In practice, all uncertainties are a combination of the two.

Systematic errors

A **systematic error** biases measurements in the same direction; the measurements are always too large or too small. If you use a metal ruler to measure length on a very hot day, all your length measurements will be too small because the metre ruler expanded in the hot weather. If you use an ammeter that shows a current of 0.1 A even before it is connected to

Learning objectives

- Distinguish between random and systematic uncertainties.
- Work with absolute, fractional and percentage uncertainties.
- Use error bars in graphs.
- Calculate the uncertainty in a gradient or an intercept.

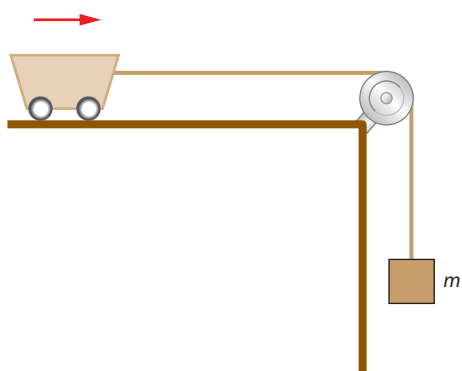


Figure 1.1 The falling block accelerates the cart.

a circuit, every measurement of current made with this ammeter will be larger than the true value of the current by 0.1 A.

Suppose you are investigating Newton's second law by measuring the acceleration of a cart as it is being pulled by a falling weight of mass m (Figure 1.1). Almost certainly there is a frictional force f between the cart and the table surface. If you forget to take this force into account, you would expect the cart's acceleration a to be:

$$a = \frac{mg}{M}$$

where M is the constant combined mass of the cart and the falling block.

The graph of the acceleration versus m would be a straight line through the origin, as shown by the red line in Figure 1.2. If you actually do the experiment, you will find that you do get a straight line, but not through the origin (blue line in Figure 1.2). There is a negative intercept on the vertical axis.

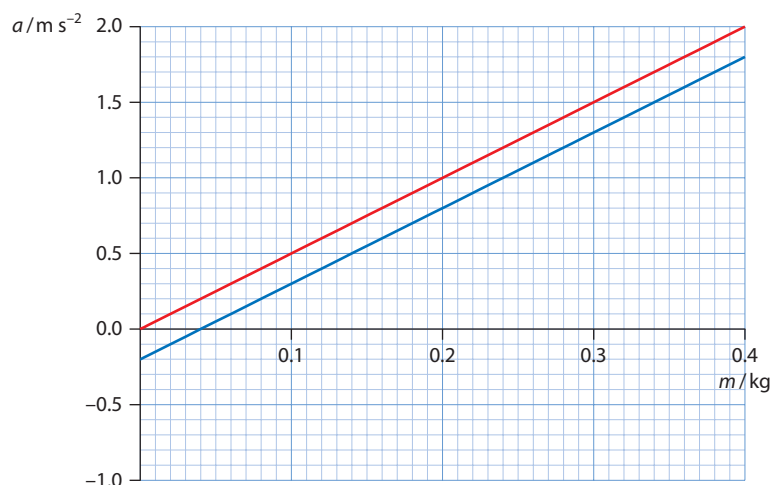


Figure 1.2 The variation of acceleration with falling mass with (blue) and without (red) frictional forces.

This is because with the frictional force present, Newton's second law predicts that:

$$a = \frac{mg}{M} - \frac{f}{M}$$

So a graph of acceleration a versus mass m would give a straight line with a negative intercept on the vertical axis.

Systematic errors can result from the technique used to make a measurement. There will be a systematic error in measuring the volume of a liquid inside a graduated cylinder if the tube is not exactly vertical. The measured values will always be larger or smaller than the true value, depending on which side of the cylinder you look at (Figure 1.3a). There will also be a systematic error if your eyes are not aligned with the liquid level in the cylinder (Figure 1.3b). Similarly, a systematic error will arise if you do not look at an analogue meter directly from above (Figure 1.3c).

Systematic errors are hard to detect and take into account.

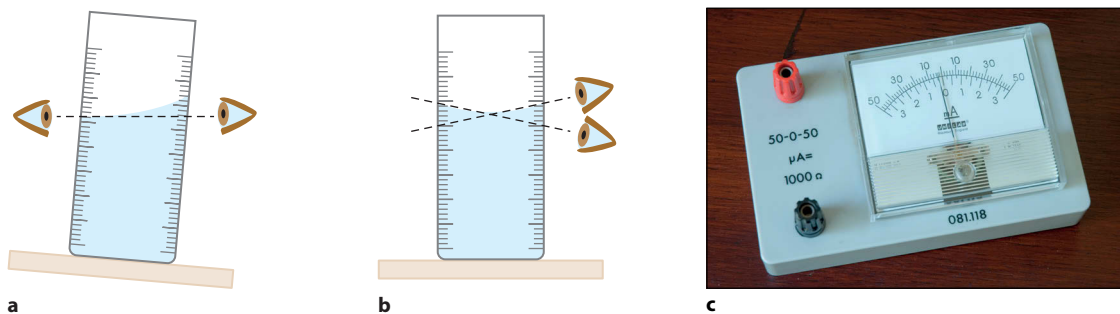


Figure 1.3 Parallax errors in measurements.

Random uncertainties

The presence of **random uncertainty** is revealed when repeated measurements of the same quantity show a spread of values, some too large some too small. Unlike systematic errors, which are always biased to be in the same direction, random uncertainties are unbiased. Suppose you ask ten people to use stopwatches to measure the time it takes an athlete to run a distance of 100 m. They stand by the finish line and start their stopwatches when the starting pistol fires. You will most likely get ten different values for the time. This is because some people will start/stop the stopwatches too early and some too late. You would expect that if you took an average of the ten times you would get a better **estimate** for the time than any of the individual measurements: the measurements **fluctuate** about some value. Averaging a large number of measurements gives a more **accurate** estimate of the result. (See the section on accuracy and precision, overleaf.)

We include within random uncertainties, **reading** uncertainties (which really is a different type of error altogether). These have to do with the precision with which we can read an instrument. Suppose we use a ruler to record the position of the right end of an object, **Figure 1.4**.

The first ruler has graduations separated by 0.2 cm. We are confident that the position of the right end is greater than 23.2 cm and smaller than 23.4 cm. The true value is somewhere between these bounds. The average of the lower and upper bounds is 23.3 cm and so we quote the measurement as (23.3 ± 0.1) cm. Notice that the uncertainty of ± 0.1 cm **is half the smallest width** on the ruler. This is the conservative way of doing things and not everyone agrees with this. What if you scanned the diagram in **Figure 1.4** on your computer, enlarged it and used your computer to draw further lines in between the graduations of the ruler. Then you could certainly read the position to better precision than the ± 0.1 cm. Others might claim that they can do this even without a computer or a scanner! They might say that the right end is definitely short of the 23.3 cm point. We will not discuss this any further – it is an endless discussion and, at this level, pointless.

Now let us use a ruler with a finer scale. We are again confident that the position of the right end is greater than 32.3 cm and smaller than 32.4 cm. The true value is somewhere between these bounds. The average of the bounds is 32.35 cm so we quote a measurement of (32.35 ± 0.05) cm. Notice

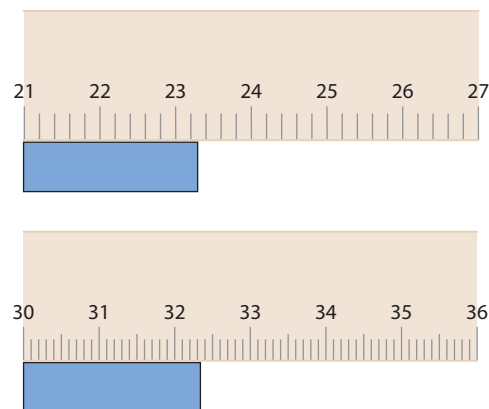


Figure 1.4 Two rulers with different graduations. The top has a width between graduations of 0.2 cm and the other 0.1 cm.